**Comment**

[in Natural Gaussian units] So with a single particle Hamiltonian in regular Quantum Mechanics, we can add an EM field by the replacement:



(and don’t need to add scalar potential φ(x) to H if in temporal gauge) For multiple particles, then we’d have:



and in 2nd quantized notation:



where **A**(**r**,t) can be any function of interest. And of course we recall **B** = ∇×**A**, and **E** = -∂**A**/∂t (again if in temporal gauge). A conventional EM wave can be represented by:



(ωk = ℏkc if units restored) and then,



A purely E field (though homogeneous) can be represented by taking k → 0. And a time-independent E field by taking ω → 0. A time-independent B field could be represented by using one of those formulas, like,



For more complicated fields, we’d probably have to step out of the Coulomb/temporal gauge, and use the full A = (φ, **A**) space-time vector potential. So how do we compare to the case when we use a quantized **A** [using Cond Mat phase convention on creation/annihilation operators – see last file]?



I guess we’d be working with:



I imagine **E** and **B** are still related via those gauge formulas above: **B** = ∇×**A**, and **E** = -∂**A**/∂t. But now x is not an operator here, rather it is a parameter, specifying all the (spatial) d.o.f. of the photon quantum field. An EM field of a particular given frequency would just be written,



or maybe could say, using the QFT phase convention:



All that’s left is to specify a field strength, and I guess this would be done by doing our calculations in the photon field eigenstate, |Φ>, such that:



and A = E/ωk. So given an E, we’d have A, and then just fix the occupation number to match, i.e.,



Or in the more general case,



So we have:



So we can fix A’s magnitude by fixing the occupation numbers of |Φ>. But now when we couple it to other particles, I think we still have to promote the x in A(x) to an operator, . Otherwise, in the H below for instance, A(x) would commute with p, and the particle’s motion would be unaffected.



So for single particle, we’d make replacement [back to Cond Matt phase convention I guess],



For multiple particles, then we’d have:



and in 2nd quantized notation:



Of course the <i| and |j> would slide past the akλ, a†kλ operators in photon space. And to recap, we do not explicitly choose the magnitude of A to be whatever, like we do in the ‘semi-classical’ picture, rather, in the fully quantized picture, we’d choose the occupation numbers of the photon eigenstate |Φ> to be what matches, at least on average, the classical picture.